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Non-sample based parameters design for system performance reliability improvement †

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Abstract

Design methods for quality generally help to improve quality over time, but do not consider change of system performance over time, resulting from degradation in components. As design methods for quality over time (performance reliability), which minimizes effects of unavoidable component degradations as well as component variations on system performance change, system model-based sampling methods using Monte-Carlo simulations have been used. But, there are main concerns related to computational efficiency and optimization in applying the sampling methods. To overcome the concerns, we propose a non-sample method for quality over time. Based on the proposed method, the process of allocating design parameters, which could minimize the noise effects with the consequence that both quality and performance reliability are optimized, is discussed. Reliability metrics such as mean time to failure and standard deviation of time to failure are optimized simultaneously for reliability improvement. Desirability functions for the metrics are introduced to perform the simultaneous optimization. The proposed method is applied to an electrical system design and compared to a sampling based design method.

***Keywords*:** Component degradation; Design parameter allocation; Quality over time; System model; System performance degradation

1. Introduction

Degradation in components due to the effects of environmental conditions produces time-variant performance in component and system with the consequence that system performance varies from the initial over time [1, 2]. What customers notice is a change in quality or performance characteristics over time [1]. Customer satisfaction is more quality-driven than value- or price-driven. And thus, customer satisfaction is a function of both quality and overall customer expectation of quality [3]. By quality we mean conformance to specifications [4, 5]. Quality over time is often referred to as performance reliability, wherein soft failure indicates that performance measures of a system or component do not conform to its performance specifications. Experience points to the belief that a product of high quality when placed in service might be a reliable product (capable of releasing performance degradation over time). However, it is not enough to consider its performance change only due to the variation in component during the manufacturing process. Design methods for high quality generally help to improve performance reliability, but change of system performance over time resulting from degradation in component is not considered in these methods. Therefore, design methods capable of diminishing unavoidable effects of component degradation on system performance have a potential to provide important performance reliability improve-

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ment, which would not be possible by the design methods for quality alone [6-8]. Thus, design methods, which take into account degradation as well as variation in component, are a critical path to customer satisfaction.

To date, the traditional design methods for system performance reliability improvement using component degradation data have been based on a sampling-based approach that uses Monte-Carlo simulation (MCS) to predict system reliability [9, 10]. More specifically, the sampling approach takes samples of the component distributions at time zero and traces their paths using their particular degradation function to provide time-variant system responses. Through tracking and comparing the time variant responses with critical specification limits, a system performance reliability function is predicted. Then, for reliability improvement, time-related robustness measures from the predicted reliability function, such as mean time to soft failure (MTTSF) and standard deviation of time to soft failure (SDTTSF), are optimized to allocate design parameters. However, there are the usual concerns when applying the sampling approach to predict reliability measures. For example, a large number of simulations and a vast computer memory are required to achieve a reasonable accuracy. Typically, $10^{n+2} \sim 10^{n+3}$ samples are needed to compute accurately a probability of failure of 10^{-n} [11]. Moreover, when variations in environmental conditions are introduced, computational load is very serious since a reliability measure for random environmental conditions is evaluated by using a weighted or simple average of all predicted reliability measures for each sampled environmental condition [10]. In addition, the sampling approach is not ideally suited for gradient-based optimization. In general, the MCS-based gradient estimation method is often computationally inefficient. Thus, other gradient approximation methods such as orthogonal array sampling-based estimation method or stochastic approximation method are required to improve computational efficiency [12]. Due to these concerns, up to now the sampling-based design approach is practical for only parameter design.

Several non-sampling approaches have been investigated to overcome the concerns in the sampling approach. Most common are the stress-strength interference (SSI) models [13]. A different approach combines time-variant first-order reliability methods (FORM) to evaluate out-crossing rate and numerical integration to build the reliability function [11]. The method has been applied to a single, larger-is-best, response of a structural system in the area of civil engineering. There are no applications to general multi-response systems. However, many industrial products are characterized by more than one measurable quantity (response). In addition, the reliability level at a time $t = 0$ is assumed to be unity, although this is not reasonable for mass-produced products. Further, these non-sampling approaches have been only used in the area of reliability analysis, not reliability design. Concerns in both the sampling approach and non-sampling approach require a new non-sampling approach capable of predicting, and improving performance reliability of multi-response engineering systems by design.

In this paper, design optimization problems that are based on a non-sample approach are formulated to allocate optimal design parameters of multiple response system for performance reliability improvement. As reliability metrics to optimize, both MTTF and SDTTSF are considered. In Sec. 2 system lifetime distribution of failure is modeled to evaluate these reliability metrics. The proposed parameter design method for reliability improvement is introduced in Sec. 3. Then the proposed design method is applied to an electrical system design in Sec. 4.

2. System lifetime distribution modeling using component degradations

2.1 Time-variant limit-state function

Let us extract from the components a vector of random design variables denoted as $\mathbf{V} = [V_1, V_2, \ldots V_m]$. These may be dimensions, resistances, spring constants and so forth. Let p be the design parameter vector comprising, for example, means and standard deviations that characterize $\mathbf{V}$, then $\mathbf{p} = [\mu_1, \mu_2, \ldots \mu_m, \sigma_1, \sigma_2, \ldots \sigma_m]$.

Component degradation models are assumed to be known along with the initial design parameter vector $\mathbf{p}$. For efficient numerical probability evaluations, the degradation models are transformed into standard normal variables. The Rosenblatt transformation, called **u-v** mapping, takes samples of the *m* arbitrarily distributed random variables (denoted as **v**) and maps them to a standard normal vector **u** comprising *m* standard normal variables [14]. The transformation is denoted by the general implicit form $\Gamma(\mathbf{u}, \mathbf{v}, \mathbf{p}) = 0$. Any nonlinear **u-v** transformations may require an iterative scheme such as Newton-Raphson. Herein we use the inverse transformation denoted as the explicit functional form $\mathbf{v} = f(\mathbf{p}, \mathbf{u})$. For a degradation distribution model we replace V with $X(t)$ and use the transformation $\Gamma(u, x(t), \mathbf{p}'(t)) = 0$. It follows that $x(t) = f(\mathbf{p}'(t), u)$. In general, $\mathbf{p}'(t)$ is given as a function of $\mathbf{p}$ and t. Thus, a sample of $X(t)$ has a functional form.

$$x(t) = f(\mathbf{p}, u, t) \tag{1}$$

For a degradation path model, $X(t)$ is in general given as a function of a coefficient vector $\mathbf{\Theta}$ and t. However, $\mathbf{\Theta}$ is determined from V and the **u**-**v** mapping provides $v = f(\mathbf{p}, u)$. It follows that $\mathbf{\Theta}$ is a function of $\mathbf{p}$ and u. Thus, a sample of $X(t)$ has a functional form

$$x(t) = f(\mathbf{\Theta}(\mathbf{p}, u), t) \tag{2}$$

Now, we have the unified general model for two different degradation models (denoted as Eqs. (1) and (2)) in **u**-space as

$$x(t) = f(\mathbf{p}, \mathbf{u}, t) \tag{3}$$

Example:

To see how we obtain a unified degradation model, let us consider a system of two components $C^{\#1}$ and $C^{\#2}$. Let V_1 and V_2 be the design variables within the components. For component $C^{\#1}$, let V_1 be normal with mean μ_1 and standard deviation σ_1, and let the degradation model be of a degradation distribution model such that we have the general mapping

$$x_1(t) = \mu_1'(t) + \sigma_1'(t)u_1$$

where $\mathbf{p} = [\mu_1\ \sigma_1]$, $\mathbf{u} = u_1$, and the particular functions are given, for example as $\mu_1'(t) = \mu_1(1 + k_1 t^2)$ and $\sigma_1'(t) = \sigma_1(1 + k_2 t^2)$, where both k_1 and k_2 indicate degradation rates. Next, following the lead in reference [9] for component $C^{\#2}$ let V_2 be normal with mean μ_2 and standard deviation σ_2, and let the degradation model be of a "degradation path" model such that the coefficient vector is $\mathbf{\Theta} = [V_2\ kV_2]$ where k represents a degradation rate. Herein coefficients of the linear path model are $\Theta_0 = V_2$ and $\Theta_1 = kV_2$; hence $X_2(t) = V_2 + kV_2 t$. For the unified degradation model of $x_2(t)$, the **u**-**v** mapping to v_2 is simply given as $v_2 = \mu_2 + \sigma_2 u_2$, and thus the final mapping is

$$x_2(t) = (\mu_2 + \sigma_2 u_2)(1 + kt)$$

where $\mathbf{p} = [\mu_2\ \sigma_2]$, $\mathbf{u} = [\ u_2\ ]$. Note that both mappings follow the general form of Eq. (3).

Now, the i^{th} time-variant system response $z_i(\mathbf{x}(t))$ has a form in terms of unified degradation models $\mathbf{x}(t) = f(\mathbf{p}, \mathbf{u}, t)$ as

$$z_i(\mathbf{x}(t)) = z_i(\mathbf{p}, \mathbf{u}, t) \tag{4}$$

Let us relate the response to a specification limit by a limit-state function of the form

$$g_i(\mathbf{x}(t)) = \pm\{z_i(\mathbf{p}, \mathbf{u}, t) - \zeta\} \tag{5}$$

where z_i is a response and ζ is either a lower or an upper specification. (Note: For upper specifications, the negative of the right side of Eq. (5) is used.) For any limit-state function, we define

$g(\mathbf{x}(t)) > 0$, $\mathbf{x}(t) \in$ Conformance region (Success region, S)
$g(\mathbf{x}(t)) = 0$, $\mathbf{x}(t) \in$ Limit-state surface (LSS)
$g(\mathbf{x}(t)) < 0$, $\mathbf{x}(t) \in$ Non-conformance region (Failure region, F)

2.2 Lifetime distribution modeling of soft failure

Based on a series system reliability concept, the system performance reliability can be interpreted as the probability that all time-variant responses satisfy their critical limits before time t [2, 15]. Thus, for m time-variant limit-state functions, system performance reliability $R_T^p(\mathbf{p}, t_L)$ at time t_L under specified component degradations $\mathbf{x}(t)$ for design parameter vector considered at design stage can be expressed as

$$R_T^p(\mathbf{p}, t_L) = \Pr\left\{\bigcap_{i=1}^m [g_i(\mathbf{p}, \mathbf{u}, \tau) > 0], \text{ for } \forall\tau \in [0, t_L]\right\} \tag{6}$$

To evaluate the system performance reliability function numerically, we apply a finite time difference method with a fixed time step h and a time index denoted as l where $l = 0, 1, \ldots L$ then $t_l = l \times h$ is the time at the l-th step, and t_L is equal to $L \times h$. Now the performance reliability function can be approximated as

$$R_T^p(\mathbf{p}, t_L) \approx \Pr\left\{\bigcap_{l=0}^L \left[\bigcap_{i=1}^m (g_i(\mathbf{p}, \mathbf{u}, t_l) > 0)\right]\right\} \tag{7}$$

And the cumulative distribution function of time to non-conformance can be written as

$$\begin{aligned} F_T^p(\mathbf{p},\ t_L) &= 1 - R_T^p(\mathbf{p},\ t_L) \\ &\approx \Pr\left\{\bigcup_{l=0}^{L}[\bigcup_{i=1}^{m}(g_i(\mathbf{p},\ \mathbf{u},\ t_1) > 0)]\right\} \end{aligned} \tag{8}$$

Then, let us define $E_{l,i}$; an *instantaneous failure* region of the i^{th} limit-state function at any selected discrete time t_l is defined as

$$E_{l,i} = \{\mathbf{u} \in \mathbf{U} : g_i(\mathbf{p},\ \mathbf{u},\ t_l) \le 0\} \tag{9}$$

and $\mathbf{E}_l$, a *system* instantaneous failure region at time t_l as

$$\mathbf{E}_l = E_{l,1} \cup E_{l,2} \cup \cdots \cup E_{l,n} = \bigcup_{i=1}^{n} E_{l,i} \tag{10}$$

From reference [16], system incremental failure probability from time t_l during time interval h is written as

$$\Delta F(\mathbf{p},\ t_l) = \Pr(\mathbf{E}_{l+1} \cup \mathbf{E}_l) - \Pr(\mathbf{E}_l) \tag{11}$$

The probability in Eq. (11) is easily evaluated by Monte-Carlo simulation using the complete limit-state function or for a good second-order approximation; wherein only pairs of intersections are invoked, we order probabilities in decreasing order, and denote these individual failure sets as $\Pr(E_{l,i}^o)$. Now, the first term on the right side of Eq. (11) is rewritten as an upper bound

$$\Pr_U(\mathbf{E}_{l+1} \cup \mathbf{E}_l) = \sum_{i=1}^{2n}\Pr(E_{l,i}^o) - \sum_{i=2,j<i}^{2n}\max\left(\Pr(E_{l,i}^o \cap E_{l,j}^o)\right) \tag{12}$$

and the second term as a lower bound

$$\begin{aligned} \Pr_L(\mathbf{E}_l) &= \Pr(E_{l,1}^o) \\ &+ \sum_{i=2}^{n}\max\left(\left[\Pr(E_{l,i}^o) - \sum_{j=1}^{i-1}\Pr(E_{l,i}^o \cap E_{l,j}^o)\right],\ 0\right) \end{aligned} \tag{13}$$

We now have the conservative approximation of Eq. (11)

$$\Delta F(\mathbf{p},\ t_l) \cong \Pr_U(\mathbf{E}_{l+1} \cup \mathbf{E}_l) - \Pr_L(\mathbf{E}_l)$$

The cumulative distribution function at time t_L is evaluated as

$$F(\mathbf{p}, t_L) = \Pr(\mathbf{E}_0) + \sum_{l=0}^{L-1}\left(\Delta F(t_l)\right) \tag{14}$$

where the first term on the right represents the non-conformance (e.g., quality) at time zero. In this paper, Eq. (14) is evaluated by using FORM (first-order reliability method) and second-order bounds on union probability whose detailed explanations are shown in the reference [16].

3. Parameter design for reliability improvement

3.1 MTTF and SDTTSF modeling

From classical reliability approaches, MTTSF (μ_T) and SDTTSF (σ_T) conditional on $F_T^P(t_0) = 0$ are [17]

$$\mu_T = \int_0^\infty t f_T^P(t)dt \tag{15}$$

$$\sigma_T = \sqrt{\int_0^\infty (t - \mu_T)^2 f_T^P(t)dt} \tag{16}$$

As an approximation to $f_T^P(t_l)$, $\underline{f}(t_l)$ is evaluated using $\underline{F}(t_l)$ for a time step size h as

$$\underline{f}(t_l) = \frac{\underline{F}(t_{l+1}) - \underline{F}(t_l)}{h} \quad \text{for } l = 1, 2, \ldots L\text{-}1 \tag{17}$$

where $\underline{F}(t_l)$ is evaluated as $F(t_l)$ in Eq. (14).

For a case that the probability of failure at $t_0 = 0$ is not zero (i.e. $F(t_0) \neq 0$), $f(t_0)$ is approximated as

$$\underline{f}(t_0) \approx \frac{\underline{F}(t_0) + \{\underline{F}(t_1) - \underline{F}(t_0)\}}{h} = \frac{\underline{F}(t_1)}{h} \tag{18}$$

We obtain approximated μ_T and σ_T in our notations by substituting $\underline{f}(t_l)$ into $f_T^P(t_l)$ in both Eq. (15) and (16) as

$$\mu_T(\mathbf{p}) \cong \sum_{l=1}^{L}\left(t_l \underline{f}(\mathbf{p}, t_{l-1})h\right) \tag{19}$$

$$\sigma_T(\mathbf{p}) \cong \sqrt{\sum_{l=1}^{L}\left((t_l - \mu_T(\mathbf{p}))^2 \underline{f}(\mathbf{p}, t_{l-1})h\right)} \tag{20}$$

There would be two possible errors in approximating μ_T and σ_T as Eq. (19) and (20). The first error would occur when the proposed method approximates continuous time events into discrete time event in evaluating cumulative distribution functions. The second error would occur when we apply FORM methods. To eliminate the first error, the time step size to satisfy the correlation coefficient condition for two consecutive limit-state functions during the time step is chosen in the reference [18]. For the second error, the error was negligible when we compared the cumulative distribution function evaluated using FORM method to one using MCS.

3.2 Formulation of parameters design problem

Formulations of optimization problems are investigated to solve the design problems through allocating desired nominal values of design variables with fixed tolerances. Thus, for parameter design we have design parameters $\mathbf{p} = [\mu_1,\dots \mu_m]$. The parameters design problem to maximize μ_T and minimize σ_T simultaneously becomes a multi-objective optimization problem. A multi-objective problem can be transformed into the standard problem by either assigning weighting factors to different objective functions or using functional transformations of the objective functions to combine the functions into a single objective function. In addition, when one objective function can be chosen as the primary, or most important objective function, and bounds or targets can be defined on all other objective function values, the primary objective function is maximized or minimized, as desired, subject to appropriate constraints on all other objective function values. A desirability function approach based on functional transformations would be a popular method. The approach assigns a "score" to a set of objective function values, and chooses parameters that maximize that score and thus provide the most desirable objective function values. The desirability function assigns numbers between 0 and 1 to the possible objective function values wherein a completely undesirable objective function value is assigned with the value 0, and a completely desirable or ideal objective function value with the value 1. In this paper, the desirability function approach proposed by Derringer and Suich is used [19].

The desirability function d_1 to maximize μ_T is written as

$$d_1(\mu_T(\mathbf{p})) = \begin{cases} 0.0 & \text{if } \mu_T(\mathbf{p}) \le L_1 \\ \dfrac{\mu_T(\mathbf{p}) - L_1}{T_1 - L_1} & \text{if } L_1 \le \mu_T(\mathbf{p}) \le T_1 \\ 1.0 & \text{if } T_1 \le \mu_T(\mathbf{p}) \end{cases} \quad (21)$$

where L_1 is a lower value for μ_T, and T_1 is the target value. The desirability function d_2 to minimize σ_T with the target value T_2 and the upper value U_2 for σ_T has the form

$$d_2(\sigma_T(\mathbf{p})) = \begin{cases} 1.0 & \text{if } \sigma_T(\mathbf{p}) \le T_2 \\ \dfrac{\sigma_T(\mathbf{p}) - U_2}{T_2 - U_2} & \text{if } T_2 \le \sigma_T(\mathbf{p}) \le U_2 \\ 0.0 & \text{if } U_2 \le \sigma_T(\mathbf{p}) \end{cases} \quad (22)$$

These values L_i and T_i in the desirability functions are set up according to design goals. The functions d_1 and d_2 are shown in Fig. 1.

The individual desirability functions are then combined using the geometric mean, which gives the overall desirability function D:

$$D(\mathbf{p}) = (d_1 \cdot d_2)^{1/2} \quad (23)$$

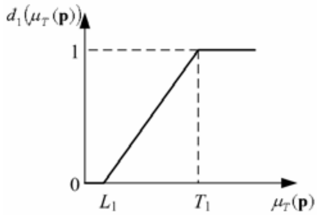
Rather than the arithmetic mean, the geometric mean is used to balance d_1 and d_2, as D is increased. The design parameter $\mathbf{p}$ maximizing the overall desirability function D maximizes d_1 and d_2, and thus maximizes μ_T and minimizes σ_T simultaneously. The parameter design problem can be formulated into the following optimization problem:

$$\begin{aligned} &\text{Minimize } \{-D(\mathbf{p})\} \\ &\text{Subject to bounds on } \mathbf{p} \end{aligned} \quad (24)$$

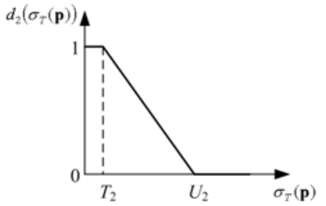
4. Case studies

A temperature control circuit's operation degrades due to impedance degradations in resistors. The optimization problem denoted as Eq. (24) is investigated to determine initial nominal values of resistors for reliability improvement of the temperature control circuit. The previous design of the temperature control circuit using a sampling-based approach [9] is compared to the proposed design.

Phadke introduced the temperature control system shown in Fig. 2 as an example of design for quality [20]. The function of a temperature control system is to maintain the temperature of a room, a bath, or some object at a target value. A temperature control system can be divided into three main modules (see Fig. 2(a)): (a) a temperature sensor (thermistor), (b) a temperature control circuit and (c) a heating element. The temperature, for example of a bath, is sensed by a thermistor, which is assumed to have a negative temperature coefficient. This means that the thermistor resistance, R_T, decreases with an increase in the tem-

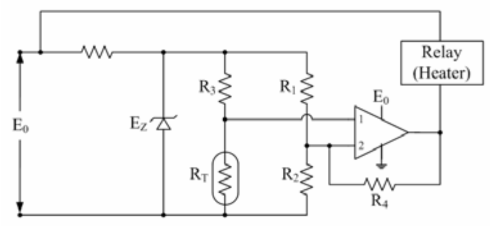


(a) Desirability function d_1 for MTTSF

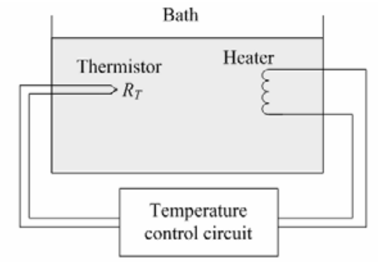


(b) Desirability function d_2 for SDTTSF

Fig. 1. Desirability functions for MTTSF and SDTTSF.



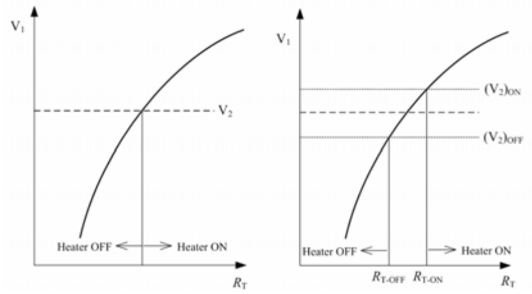
(a) Temperature control system



(b) Control circuit

Fig. 2. Temperature control system.

perature of the bath. When the bath temperature rises above a certain value, the resistance R_T drops below a threshold value so that the difference in the voltages between terminals 1 and 2 of the amplifier becomes sufficiently large and negative. This actuates the relay and turns OFF the heater, as shown in Fig. 3(a). Likewise, when the temperature falls below a certain value, the difference in voltages between the terminals 1 and 2 becomes sufficiently large and positive so that the relay is actuated and the heater is turned ON. The uncertain heat transfer parameters of the bath (for example, thermal capacity, the rate of heat



(a) (b)

Fig. 3. Operation of a temperature control circuit.

input by the heater, and the rate of heat loss to the surroundings) and the considerations of longevity of the heater (too frequent transitions of ON and OFF can greatly reduce the heater life) require hysteresis of the temperature control circuit.

The hysteresis allows a period for the heater to stay ON and the temperature to rise slightly above the target temperature, as shown in Fig. 3(b). Similarly, the hysteresis allows the heater to stay OFF for a period of time and the temperature to drop a little bit below the target temperature. Thus, the temperature control circuit provides a way of setting the target value of the resistance R_T corresponding to a target temperature, and then turns the heater ON and OFF at the corresponding specified values of $R_{T\text{-}ON}$ and $R_{T\text{-}OFF}$.

Through standard techniques of circuit analysis, the voltages in the terminals 1 and 2 can be expressed as

$$V_1 = \frac{R_T}{R_T + R_3} E_z \tag{25}$$

$$(V_2)_{ON} = \frac{R_2(E_z R_4 + E_0 R_1)}{R_1R_2 + R_2R_4 + R_1R_4} \tag{26}$$

$$(V_2)_{OFF} = \frac{R_2 R_4 E_z}{R_1R_2 + R_2R_4 + R_1R_4} \tag{27}$$

where R_1, R_2, R_3, and R_4 are impedance values of the four resistors, E_0 is the power supply voltage, and E_z the voltage of the Zener diode. Combination of impedance values provides the threshold voltages to $(V_2)_{ON}$ and $(V_2)_{OFF}$. The relay turns on when $V_1 - (V_2)_{ON} = 0$. Solving for this, we find

$$\begin{aligned} &V_1 - (V_2)_{ON} = \Delta V \\ &= \frac{R_{T-ON}}{R_{T-ON} + R_3} E_z - \frac{R_2(E_z R_4 + E_0 R_1)}{R_1R_2 + R_2R_4 + R_1R_4} = 0 \end{aligned} \tag{28}$$

If we set ΔV = 0 (the point at which the relay turns on) and solve for $R_{\text{T-ON}}$, then

$$R_{\text{T-ON}} = \frac{R_3 R_2 (\mathrm{E}_z R_4 + \mathrm{E}_0 R_1)}{R_1 (\mathrm{E}_z R_2 + \mathrm{E}_z R_4 - \mathrm{E}_0 R_2)} \quad (29)$$

As well, the relay turns off when $V_1 - (V_2)_{\text{OFF}} = 0$. Solving for this, we find

$$\begin{aligned} & V_1 - (V_2)_{\text{OFF}} = \Delta V \\ & = \frac{R_{\text{T-OFF}}}{R_{\text{T-OFF}} + R_3} \mathrm{E}_z - \frac{R_2 R_4 \mathrm{E}_z}{R_1 R_2 + R_2 R_4 + R_1 R_4} = 0 \end{aligned} \quad (30)$$

By setting ΔV = 0 (the point at which the relay turns off) and then solving for $R_{\text{T-OFF}}$, we find

$$R_{\text{T-OFF}} = \frac{R_2 R_3 R_4}{R_1 (R_2 + R_4)} \quad (31)$$

The variation in the impedances $R_{\text{T-ON}}$ and $R_{\text{T-OFF}}$, due to variation in the values of the other circuit components provides the variation in $(V_2)_{\text{ON}}$ and $(V_2)_{\text{OFF}}$, and thus causes improper circuit operation. As well, the possible impedance degradations of the fours resistors would lead the impedances $R_{\text{T-ON}}$ and $R_{\text{T-OFF}}$ to vary versus time. The target-is-best impedances $R_{\text{T-ON}}$ and $R_{\text{T-OFF}}$ that have been used as quality characteristics of the temperature control circuit are performances of interest. These two impedances comprise six independent variables comprising four control variables and two noise variables. The four control variables are the design variables V_1, V_2, V_3, and V_4 associated with the impedances of the four resistors R_1, R_2, R_3, and R_4 respectively. The two noise variables are denoted as V_5, and V_6 associated with the voltages of the power supply (E_0) and Zener diode (E_z) respectively. All six variables V_i for $i = 1,\ldots 6$ are assumed to be independently normally distributed with mean and standard deviation, μ_i and σ_i wherein $\sigma_i = tol_i/3$ with tol_i = 5% of μ_i, so that the design parameters are the means and in general $\mathbf{p} = [\mu_1, \ldots \mu_4]$ for the fixed tolerances. The bounds on the design parameters are $2 \le \mu_1 \le 6$ kΩ, $4 \le \mu_2 \le 12$ kΩ, $0.5 \le \mu_3 \le 1.5$ kΩ, $30 \le \mu_4 \le 45$ kΩ.

In this example, we assume all of the four impedances V_i for $i = 1,\ldots 4$ degrade. Following the lead in reference [9], the linear degradation path model of any impedance versus time is given as $X_i(t) = V_i(1+k_i t)$ where $k_i = 1.5\cdot 10^{-3}$. Then, the unified degradation model of any impedance versus time is written as

$$x_i(t) = \mu_i(1+k_i t) + \sigma_i(1+k_i t)u_i \quad \text{for } i = 1, \ldots 4 \quad (32)$$

Voltages of the power supply (E_0) and Zener diode (E_z) do not degrade in this example, so their degradation models are

$$x_i = v_i = \mu_i + \sigma_i u_i \quad \text{for } i = 5,6 \quad (33)$$

The time-variant responses, $z_1(t)$ for $R_{\text{T-ON}}$ and $z_2(t)$ for $R_{\text{T-OFF}}$, in terms of design parameters in **u**-space have the form

$$z_1(\mathbf{p},\mathbf{u},t) = \frac{x_3(t)x_2(t)(x_6 x_4(t) + x_5 x_1(t))}{x_1(t)(x_6 x_2(t) + x_6 x_4(t) - x_5 x_2(t))} \quad (34)$$

$$z_2(\mathbf{p},\mathbf{u},t) = \frac{x_2(t)x_3(t)x_4(t)}{x_1(t)(x_2(t) + x_4(t))} \quad (35)$$

We now investigate two parameter design cases to maximize μ_T and minimize σ_T simultaneously: these cases are a) the single response $R_{\text{T-ON}}$, and b) both responses $R_{\text{T-ON}}$ and $R_{\text{T-OFF}}$.

CASE (a): $R_{\text{T-ON}}$

Based on the work in reference [9], we set the target for $R_{\text{T-ON}}$ at 2.95 kΩ, and the lower and upper specification limits at LSL_1 = 1.9 kΩ, USL_1 = 4.0 kΩ. The two limit-state functions are then

$$\begin{aligned} & g_1(\mathbf{p},\ \mathbf{u},\ t) = 4.0 - z_1(\mathbf{p},\ \mathbf{u},\ t) \\ & g_2(\mathbf{p},\ \mathbf{u},\ t) = z_1(\mathbf{p},\ \mathbf{u},\ t) - 1.9 \end{aligned} \quad (36)$$

The feasible design $\mathbf{p_i}$ that maximizes $\underline{R}$ ($t = 0$) is $\mathbf{p_i}$ = [4, 8, 1, 40]. From engineering experience, increasing μ_T might lead to an increased σ_T, or reducing σ_T might lead to a reduced μ_T. Thus, there might exist trade-offs between μ_T and σ_T. The trade-offs are controlled by using two different sets of desirability function parameters:

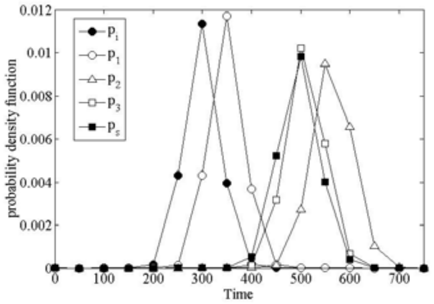
(a) L_1 = 300 and T_1 = 700 for μ_T, T_2 = 0, and U_2 = 36 for σ_T

(b) L_1 = 300 and T_1 = 700 for μ_T, T_2 = 0, and U_2 = 60 for σ_T

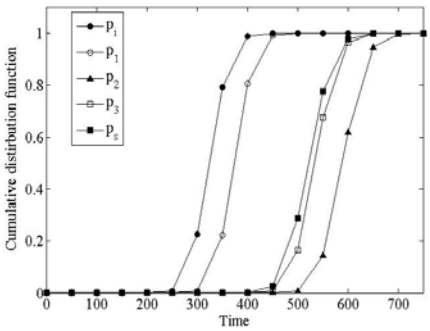
For a time step of h = 50 with $0 \le t \le 750$, results of optimal nominal values for these different sets of desirability function parameters, $\mathbf{p_1}$ for (a) and $\mathbf{p_2}$ for (b) are shown in Table 1. For a design comparison, we use the sampling-based method from reference [9] and indicate their results in Table 1 as $\mathbf{p_S}$. To show the flexibility of the proposed approach, let us add a quality constraint. That is, with (b) for desirability func-

Table 1. Parameter design results of temperature control circuits for $R_{T\text{-}ON}$.

	Nominal values				
	Feasible design	Proposed approach			Sampling approach [9]
				Constrained	
	$\mathbf{p_i}$	$\mathbf{p_1}$	$\mathbf{p_2}$	$\mathbf{p_3}$	$\mathbf{p_S}$
μ_1	4.00	2.74	2.65	2.64	4.25
μ_2	8.00	4.00	4.00	4.00	7.50
μ_3	1.00	1.50	1.18	1.26	0.95
μ_4	40.00	45.00	41.05	45.00	40.60
μ_T	349.31	398.63	614.37	559.29	546.49
σ_T	34.94	33.77	40.11	38.12	40.31



(a) Probability density function, $\underline{f}(t)$



(b) Cumulative distribution function, $\underline{F}(t)$

Fig. 4. $\underline{f}(t)$ and $\underline{F}(t)$ of temperature control circuits for $R_{T\text{-}ON}$ according to designs.

tion parameters, let us impose the condition such that $\underline{F}(t = 0) \le Y_0$. Y_o is set as $6.9\text{x}10^{-8}$ which is identical to $\underline{F}(t = 0)$ for $\mathbf{p_S}$. The new design is shown as $\mathbf{p_3}$ in Table 1. The plots of both $\underline{f}(t)$ and $\underline{F}(t)$ for each design are shown in Fig. 4. The metrics μ_T and σ_T in Fig. 4 agree with those in Table 1. Note that the metrics might vary according to time step sizes. Compared to design $\mathbf{p_i}$, design $\mathbf{p_1}$ increases μ_T by about 14% and reduces σ_T by about 3%, and the design $\mathbf{p_2}$ increases not only μ_T by about 75% but σ_T by about 15%. The design $\mathbf{p_3}$ provides a better design than design $\mathbf{p_S}$ since we have increased μ_T and decreased σ_T within the same quality level.

CASE (b): $R_{T\text{-}ON}$ and $R_{T\text{-}OFF}$

Based on the physics, we let the targets for $R_{T\text{-}ON}$ and $R_{T\text{-}OFF}$ be 2.96 and 1.67 kΩ, respectively. Their lower and upper limit specification are 20% of target, and thus lower and upper specification limits are $LSL_1 = 2.36$ kΩ, $USL_1 = 3.54$ kΩ, $LSL_2 = 1.336$ kΩ, $USL_2 = 2.004$ kΩ. The four limit-state functions are

$$\begin{aligned} g_1(\mathbf{p},\ \mathbf{u},\ t) &= 3.54 - z_1(\mathbf{p},\ \mathbf{u},\ t) \\ g_2(\mathbf{p},\ \mathbf{u},\ t) &= z_1(\mathbf{p},\ \mathbf{u},\ t) - 2.36 \\ g_3(\mathbf{p},\ \mathbf{u},\ t) &= 2.004 - z_2(\mathbf{p},\ \mathbf{u},\ t) \\ g_4(\mathbf{p},\ \mathbf{u},\ t) &= z_2(\mathbf{p},\ \mathbf{u},\ t) - 1.336 \end{aligned} \tag{37}$$

The feasible design $\mathbf{p_i}$ that maximizes $\underline{R}(t = 0)$ is $\mathbf{p_i} = [4, 8, 1, 40]$. We use two different sets of desirability function parameters:

(c) $L_1 = 100$ and $T_1 = 350$ for μ_T, $T_2 = 0$, and $U_2 = 30$ for σ_T

(d) $L_1 = 100$ and $T_1 = 350$ for μ_T, $T_2 = 0$, and $U_2 = 40$ for σ_T

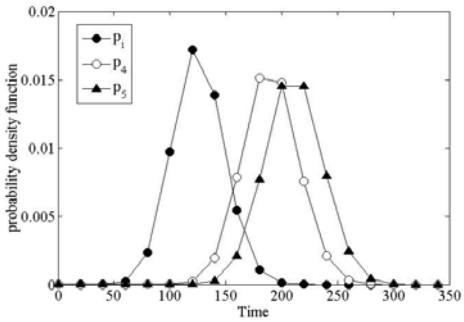
For a time step of $h = 20$ with $0 \le t \le 360$, results of optimal nominal values for these different sets of desirability function parameters, $\mathbf{p_4}$ for (c) and $\mathbf{p_5}$ for (d) are shown in Table 2. The plots of $\underline{F}(t)$ and $\underline{f}(t)$ for each design are shown in Fig. 5. Compared to the design $\mathbf{p_i}$, the design $\mathbf{p_4}$ increases μ_T and σ_T by about 44% and 8%, respectively. The design $\mathbf{p_5}$ increases μ_T and σ_T by about 58% and 14%, respectively. Therefore, the proposed design method significantly improves system performance reliability of the temperature control circuit.

5. Conclusions

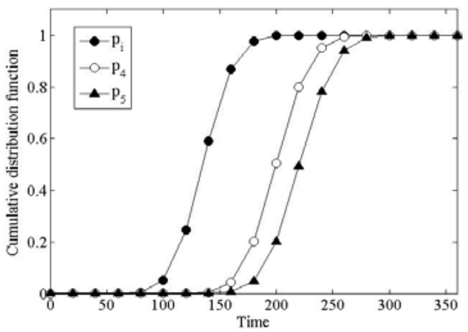
We have presented a non-sampling based parameter design method that allocates nominal values to minimize effects of component degradations as well as component variations on system performance change. These effects were evaluated in terms of desirability function values which consisted of reliability metrics, mean time to soft failure and standard deviation of time to soft failure. In the evaluation of these metrics, time-variant limit-state functions through setting up response specifications of system responses were implemented. Then these desirability

Table 2. Parameter design results of temperature control circuits for both $R_{T\text{-}ON}$ and $R_{T\text{-}OFF}$.

	Design		
	$\mathbf{p}_i$	$\mathbf{p}_4$	$\mathbf{p}_5$
μ_1	4.00	4.79	6.00
μ_2	8.00	8.74	8.04
μ_3	1.00	1.03	1.35
μ_4	40.00	39.77	39.77
μ_T	145.26	210.01	230.55
σ_T	22.81	24.72	26.10



(a) Probability density function, $\underline{f}\,(t)$



(b) Cumulative distribution function, $\underline{F}\,(t)$

Fig. 5. $\underline{f}\,(t)$ and $\underline{F}\,(t)$ according to designs for both $R_{T\text{-}ON}$ and $R_{T\text{-}OFF}$.

function values were optimized simultaneously to allocate design parameters for system reliability improvement. The proposed non-sample based design method was compared to system model-based sampling methods using Monte-Carlo simulations using the temperature control circuit design example. Moreover, two responses of the temperature control circuit were simultaneously considered for more realistic application in this paper, while a single response is considered in the sampling-based approach. The proposed non-sample based design method would overcome concerns of the sampling-based methods. Work is ongoing to extend the proposed design method to engineering systems with degraded components which lead to change in system topology as well as system performance inducing system failure.

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